On the equivalence classes of spherical curves by Reidemeister moves Ⅰ and Ⅲ

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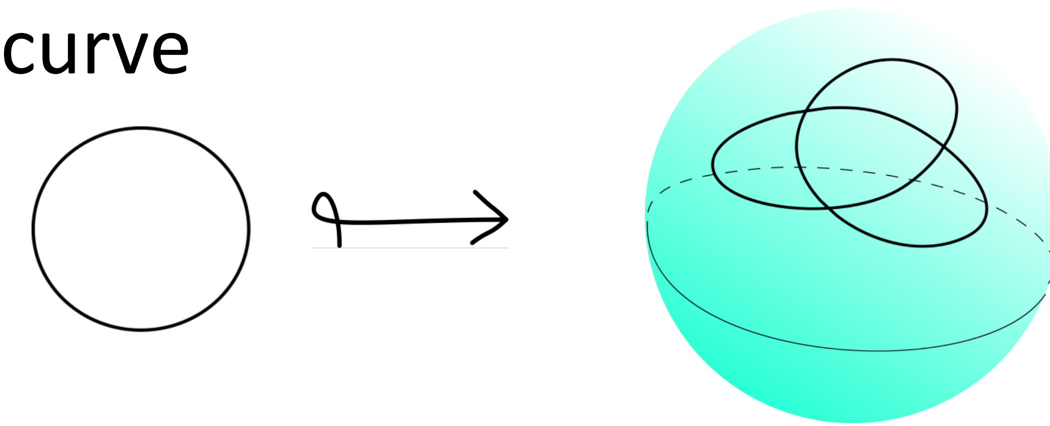
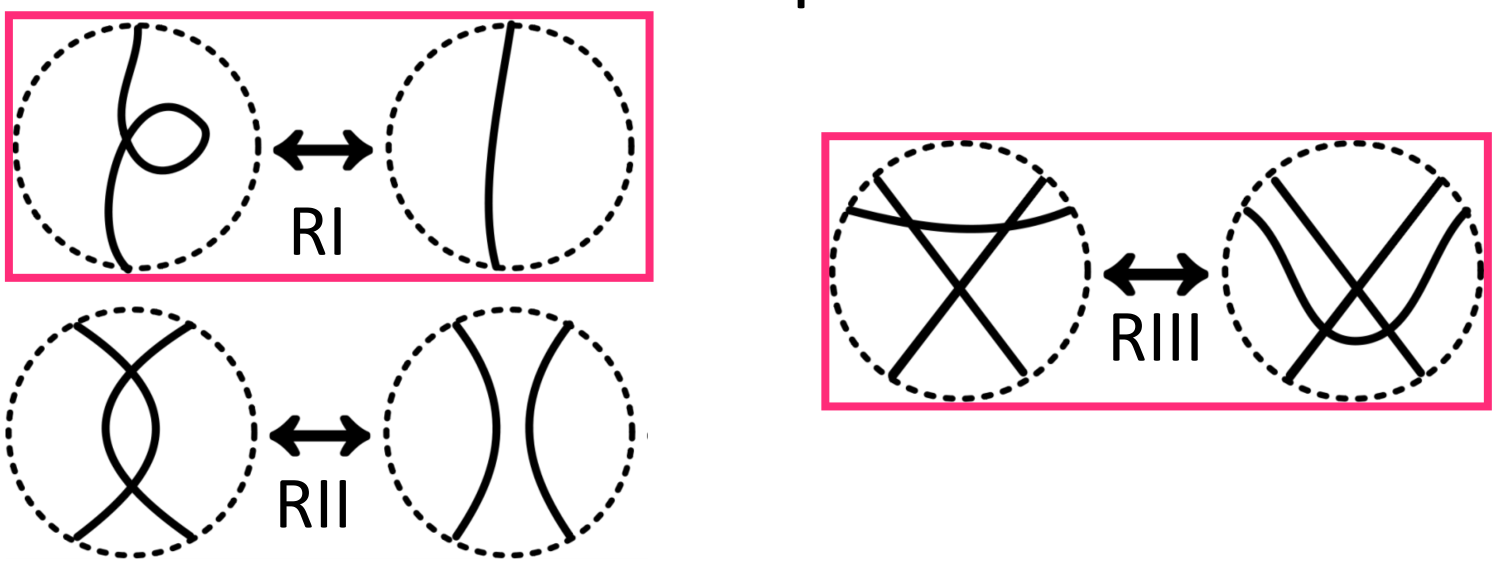
25,12,2017

Reidemeister moves of spherical curves

Spherical curve



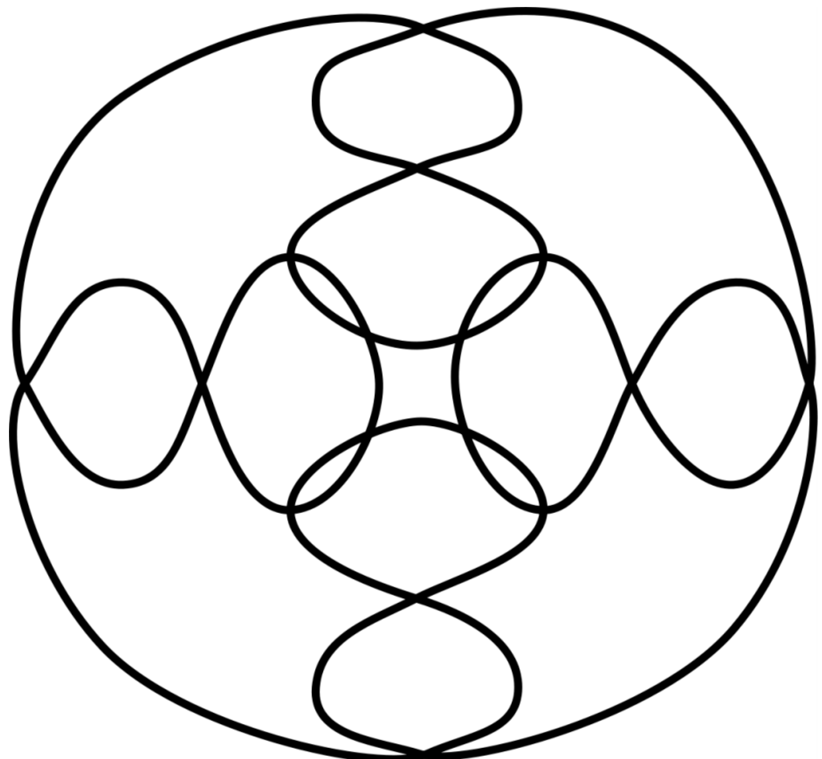
Reidemeister moves of spherical curves



In this talk, we focus on RI and RIII.

Motivation

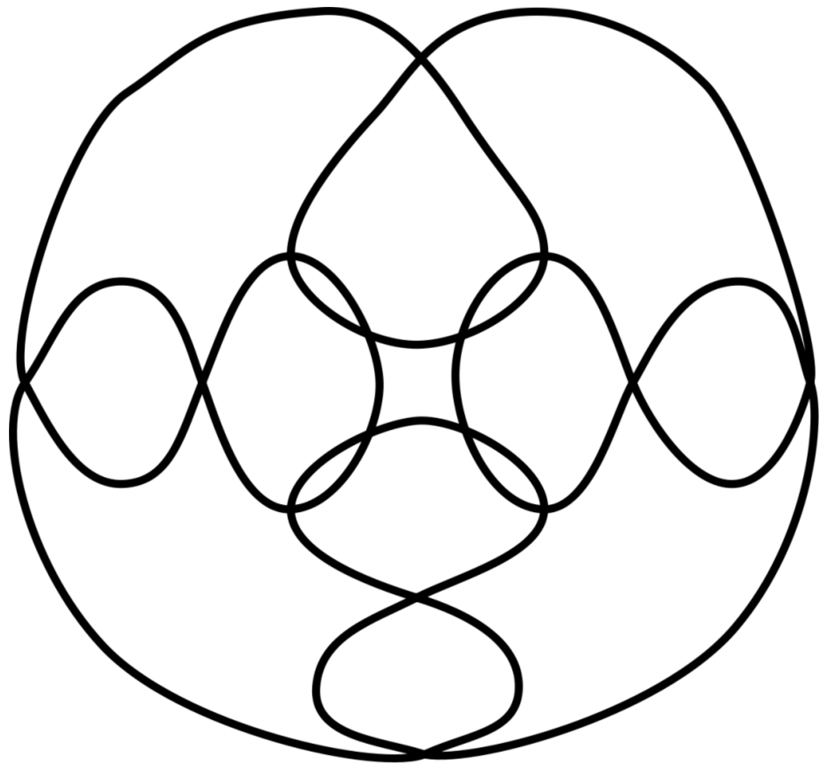
An example of Hagge-Yazinski



T. Hagge and J. Yazinski,
On the necessity of Reidemeister move 2 for simplifying immersed planar curves,
Banach Center Publ. 103 (2014), 101–110.

Motivation

An example of Ito-Takimura

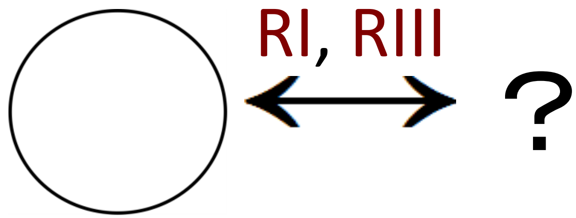


Moreover, they found an infinite number of examples!

N. Ito and Y. Takimura,
On a nontrivial knot projection under (1,3) homotopy,
Topology Appl. 210 (2016), 22-28.

Question

What spherical curve is obtained from the simple closed curve by RI, RIII and ambient isotopy?



Today's problem

P: a spherical curve

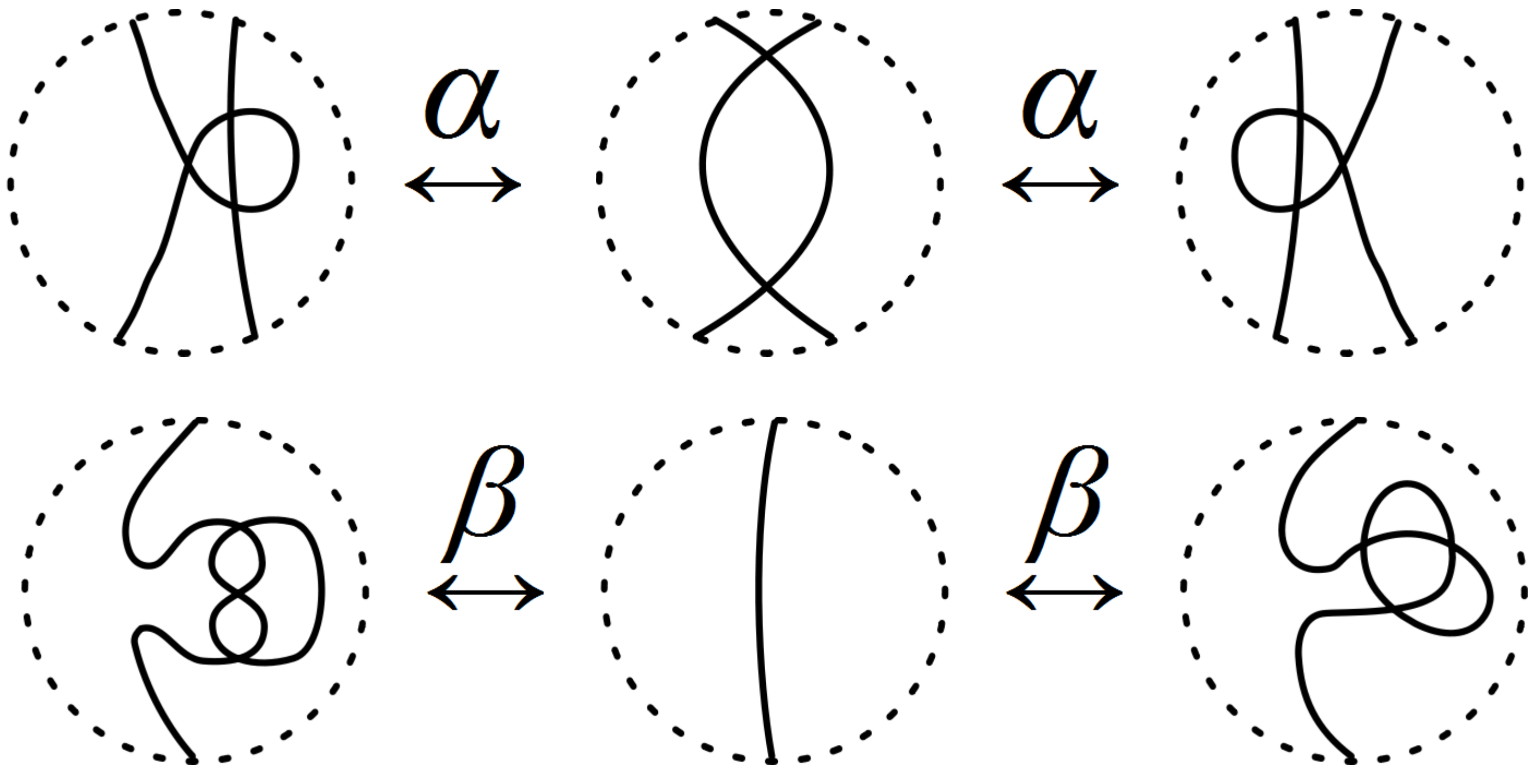
What spherical curve *P'* is obtained from *P* by some RI's, <u>a single</u> RIII and ambient isotopy?

some RI's, single RIII

$P \longleftrightarrow P'$

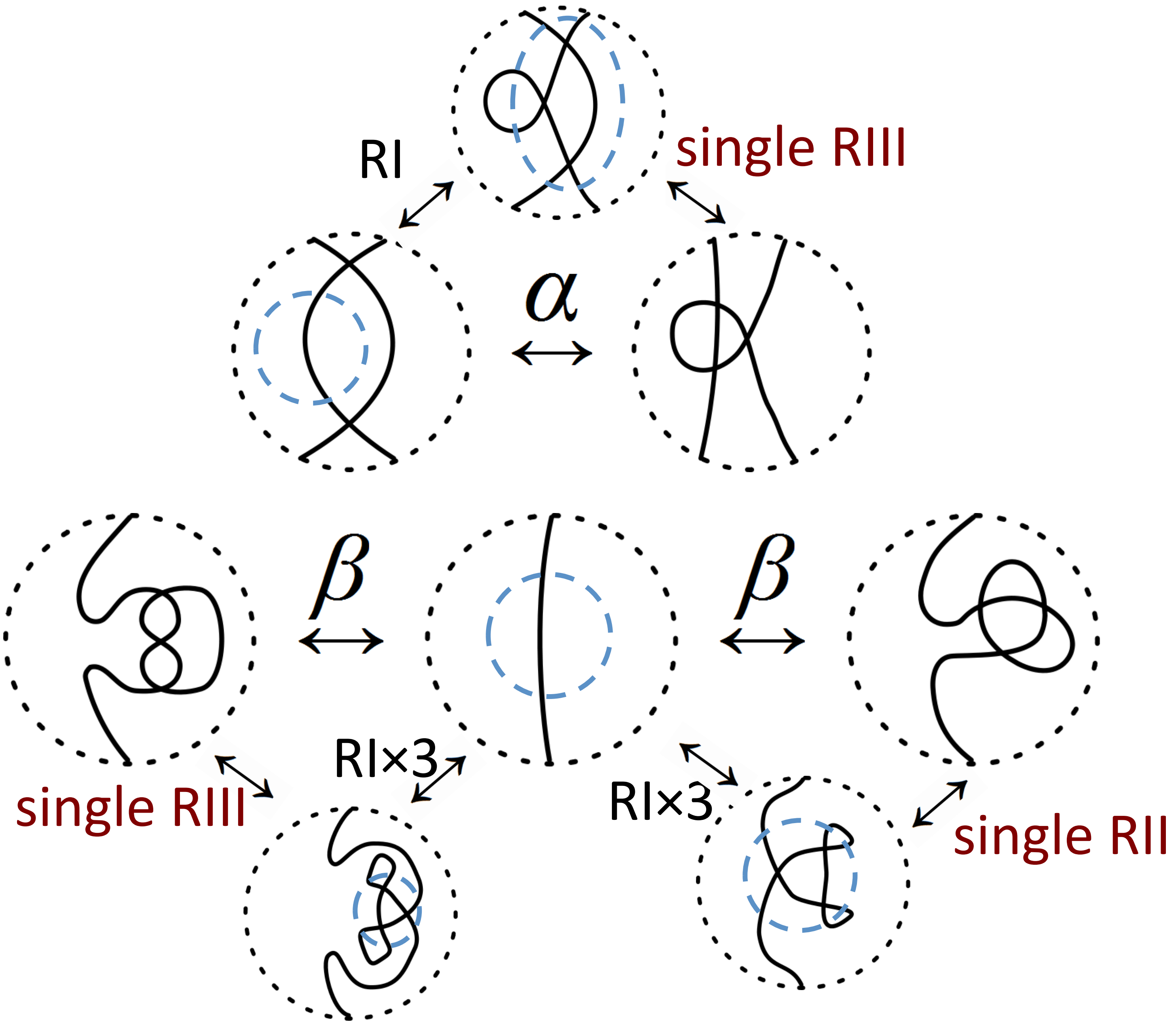
Preliminaries

Def (key operations)



※講演ではβのみ上記のように両方の操作を紹介しました．しかし，講演中に谷山公規氏(早稲田大)に頂いたコメントの通り，局所的な部分の取り替え操作なのでα, βいずれも右側のみ（もしくは左側のみ）の操作だけで十分でした．本スライド報告は操作後に得られる球面曲線が大きく異なるので両方向の操作を紹介しておきます．

Preliminaries



Preliminaries

Def (reducible)

P: a spherical curve on S^2

D_1, D_2: disks on S^2

If $^\exists E (\subset S^2)$: circle s.t.

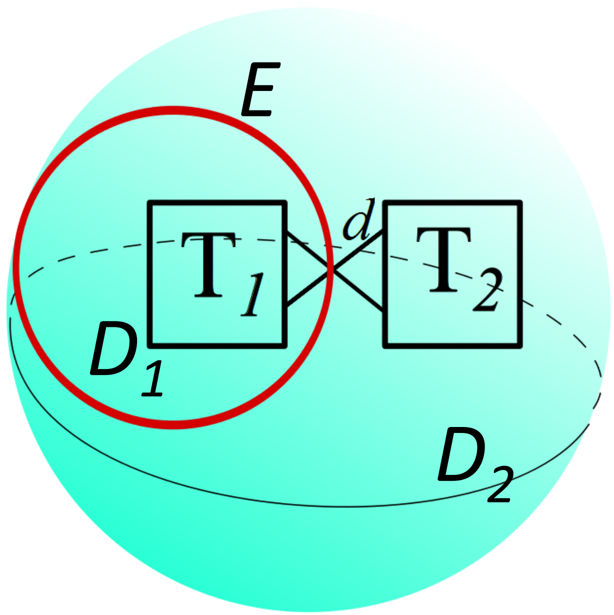
$D_1 \sqcup_E D_2 = S^2$

$E \cap P = \{$a double pt. $d\}$

$T_1 \subset D_1$, $T_2 \subset D_2$,

then we say that P is **reducible**.

If $\nexists E$, then we say that P is **irreducible**.



Main result

P: a spherical curve

n(*P*): the num. of the double points of *P*

P, *P'*: spherical curves

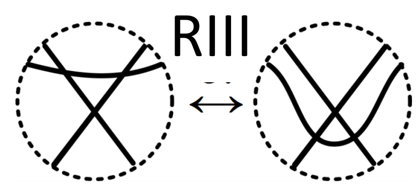
Suppose *P*, *P'*: irreducible

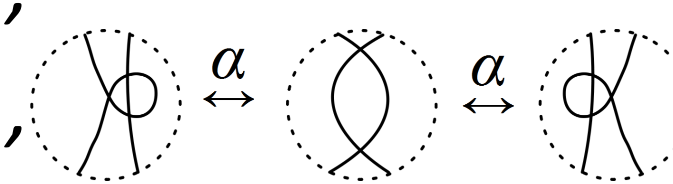
$$P \xleftrightarrow{\text{some RI, single RIII}} P'$$

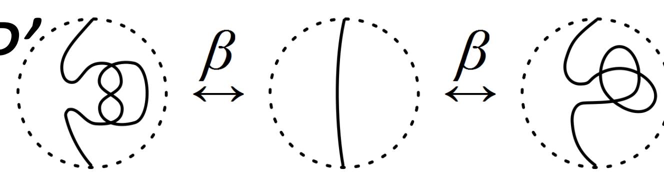
Then, |*n*(*P'*)-*n*(*P*)|= 0, 1 or 3.

In particular,

- |*n*(*P'*)-*n*(*P*)|= 0 ⇔ $P \xleftrightarrow{\text{single RIII}} P'$
- |*n*(*P'*)-*n*(*P*)|= 1 ⇔ $P \xleftrightarrow{\text{single } \alpha} P'$
- |*n*(*P'*)-*n*(*P*)|= 3 ⇔ $P \xleftrightarrow{\text{single } \beta} P'$







Complex induced by spherical curves and RIII, α, β

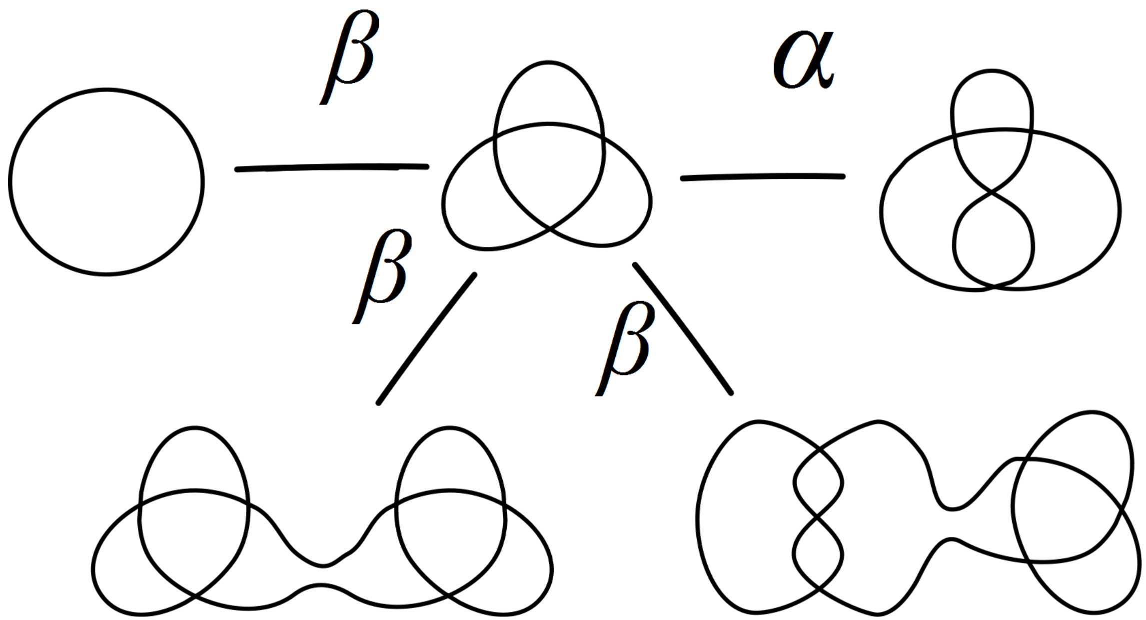
A spherical curve $P \longleftrightarrow$ vertex v_P

P, P': spherical curves

If P' is obtained from P by some RI's, a single RIII and ambi. iso.,

then there exists an edge from v_P *to* $v_{P'}$.

Complex induced by spherical curves and RIII, *α*, *β*



P, *P'*: irredu. spherical curves

$d_3(P,P')$:=min{*k*|*P'* is obtained from *P*
by some RI's, <u>*k* RIII's</u> and amb. iso.}

Corollary of main result

P, *P'*: spherical curves

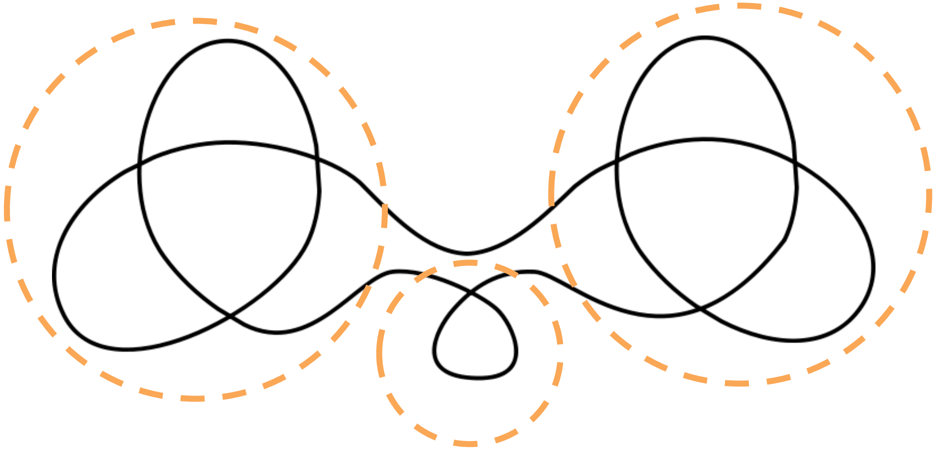
$$P \xleftrightarrow{\text{RI, RIII}} P' \quad \Leftrightarrow \quad P \xleftrightarrow{\text{RIII}, \alpha, \beta} P'$$

講演では上記を主結果の系として紹介しました．しかし講演後，早野健太氏（慶應大）から指摘・質問を頂き，当Cor. が主結果から直ちに得られるという報告は適切ではないとわかりました．
従いまして，本講演記録では当Cor. の報告を控えさせていただきます．

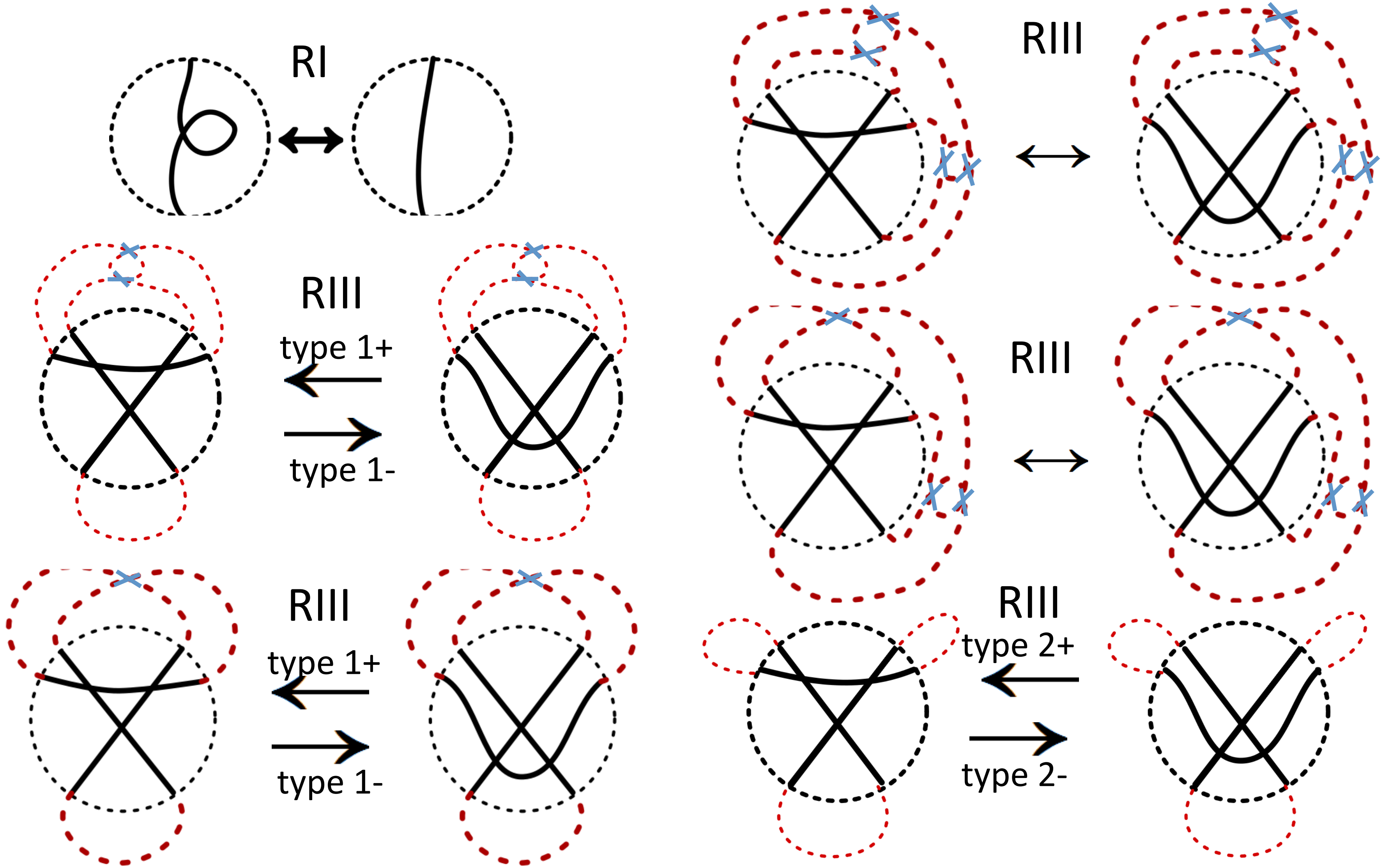
Key Lemma

P: a spherical curve

$f_c(P)$: the num. of prime factors of *P*



Key Lemma



講演では局所変形を施す部分の外側の繋がり方の場合分けが不十分でした.
講演中に瀧村祐介氏（学習院中）に頂いたコメントにより補完しました.

Key Lemma

P: a spherical curve
$n(P)$: the num. of the double points of *P*
Suppose that *P'* is obtained from *P* by RI's,
a single RIII and ambient iso.
Then we have;

$$
\begin{aligned}
f_c(P') = f_c(P) + n(P')-n(P)
&+1 \ \#\{\text{RIII} \mid \text{type } 1+\} \\
&+2 \ \#\{\text{RIII} \mid \text{type } 2+\} \\
&+(-1) \ \#\{\text{RIII} \mid \text{type } 1-\} \\
&+(-2) \ \#\{\text{RIII} \mid \text{type } 2-\}
\end{aligned}
$$