j.w. / Biao Ma

§0

$\sqrt{2} \notin \mathbb{Q}$

$\sqrt{2} \notin \mathbb{Q}(\sqrt{3})$

$\sqrt{2} \notin \mathbb{Q}(\sqrt{3}, \sqrt{5})$

prf?

$\mathrm{Gal}(\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})/\mathbb{Q}) \cong (\mathbb{Z}/2\mathbb{Z})^3$

$\leftrightsquigarrow$ br. cover of $L = L_1 \cup L_2 \cup L_3$

応用　5次以上の解の公式はない

作図問題 $\left\{ \begin{array}{l} \text{角の三等分} \\ \sqrt[3]{2} \text{ は不可} \end{array} \right.$

§1 π_L and Δ_L

$L = L_1 \cup \cdots \cup L_d \subset S^3$

d-comp link

$\pi_L := \pi_1(S^3 - L)$

$\mathrm{ab} : \pi_L \twoheadrightarrow \pi_L^{ab} = \langle t_1, \dots, t_d \rangle$

$\cong \mathbb{Z}^d$

$\ker \mathrm{ab} \leftrightsquigarrow X_\infty \to X = S^3 - L$ ($\mathbb{Z}$-cover)

$H_1(X_\infty) \cong (\ker \mathrm{ab})^{ab}$

π_L^{ab}?　$\Lambda := \mathbb{Z}[t_1^{\pm}, \dots, t_d^{\pm}]$

fin gen Λ-module

$\mathrm{ord}(H_1(X_\infty)) = (\Delta_L(t_1, \dots, t_d))$

$\left(H_1(X_\infty) \cong \Lambda/(\Delta_L) \right)$

J, L links

$$\begin{array}{ccc} \pi_J & \overset{\cong}{\longrightarrow} & \pi_L \\ {\scriptstyle ab}\downarrow & & \downarrow \\ \mathbb{Z}^d & \longrightarrow & \mathbb{Z}^d \end{array} \Rightarrow \Delta_J \overset{?}{=} \Delta_L$$

up to $GL_d\mathbb{Z}$-action

$x \longmapsto Vx$, $V \in GL_d\mathbb{Z}$

eg $\Delta_J = t_1 t_2 - 1$

$\Rightarrow \Delta_L = t_1^a t_2^b - 1$

$a, b \in \mathbb{Z}$, coprime

Q. $\hat{\pi}_L := \varprojlim_{\Gamma \triangleleft \pi_L} \pi_L/\Gamma$ 副有限完備化

$\hat{\pi}_J \cong \hat{\pi}_L \Rightarrow$?

§3. $\hat{\pi}$

π: fin gen gp

fin. pres

res. fin

$\Updownarrow$

$\exists\ \pi \hookrightarrow \hat{\pi}$

Q. $\hat{\pi} \cong \hat{\Gamma} \Rightarrow \pi \cong \Gamma$ か?

[Grothendieck 1970]

一般には No

π_1(3-mfd) : res fin. by Hempel + Perelman

Rem $\hat{\pi} \cong \hat{\Gamma} \underset{\text{iff.}}{\Longleftrightarrow}$ {π の有限商の同型類全体}

= {Γ の 〃}

unknown $\exists$? $J \neq K$ prime knots

s.t. $\hat{\pi}_J \cong \hat{\pi}_K$.

known · If J = fig. eight, then yes

etc

· geometry が一致

$\exists$ 問題集

ICMAT

Thm (U.) · $\Delta_J \doteq \Delta_K$

prf $\hat{\mathbb{Z}} := \varprojlim \mathbb{Z}/n\mathbb{Z}$

$(\Delta_J(t)) = (\Delta_K(t^\nu))$

$\exists \nu \in \hat{\mathbb{Z}}^\times$ in $\mathbb{Z}_p[[t^{\hat{\mathbb{Z}}}]]$

· 円分因子をキャンセル (NAK Lemma)

· Fried's prop (use Artin Mazur Zeta)

§3. $\hat{\pi}_J \cong \hat{\pi}_L$ のとき

$$\begin{array}{ccc} \hat{\pi}_J & \cong & \hat{\pi}_L \\ \text{ab} \downarrow & & \downarrow \\ \hat{\mathbb{Z}}^d & \cong & \hat{\mathbb{Z}}^d \\ \ast & \mapsto & V\ast \end{array}$$

$V \in GL_d \hat{\mathbb{Z}}$.

Thm $\Delta_J \doteq \Delta_L$

up to $GL_d \hat{\mathbb{Z}}$-action

Cor If $d = 2$ then

$|lk(J_1, J_2)| = |lk(L_1, L_2)|$

☺ $lk = \Delta(1.1)$

Cor If J: hyp

$GL_d \hat{\mathbb{Z}}$ を $GL_d \mathbb{Z}$ にとりかえられる。

☺ [Yi Liu 2023 Invent] の Thm 1.2

Wilton Zalesskii Agol Wise

Q -般には?

Q $d \geq 2$

Casson Lin type inv.