On Sharp Moves

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Introduction.
The crossing change is a local move for a knot diagram:

$u(K)$: the minimum number of crossing changes making $K$ into the trivial knot
**Theorem.** [M.G. Scharlemann (1985)]

\[ u(K) = 1 \implies K: \text{prime}. \]

**Example.** The trefoil knot is unknotted by a single crossing change.
The $\Delta$ move is a local move for a knot diagram:

$\Delta$ move

$u_\Delta(K)$: the minimum number of $\Delta$ moves making $K$ into the trivial knot
Example. The trefoil knot is unknotted by a single $\Delta$ move.

\[ \Delta - \text{move} \]

Question. $u_\Delta(K) = 1 \implies K: \text{prime?}$
The # move is a local move for an oriented knot diagram:

\[ u_\#(K) : \text{the minimum number of } \# \text{ moves making } K \text{ into the trivial knot} \]
Example. The trefoil knot is unknotted by a single # move.

Question. $u_\#(K) = 1 \implies K: \text{prime?}$
Background.
Theorem \cite{Murakami-Sakai1993}

$K$: trefoil knot, \quad $J_n$: 2-bridge knot $C(4,2n)$, 
$\mu'(K) = 1$

$\implies \mu'(K\#J_n) = 1$. 

\[\begin{align*}
K: \text{trefoil knot,} & \quad J_n: 2\text{-bridge knot } C(4,2n), \\
\mu'(K) = 1 & \quad \implies \mu'(K\#J_n) = 1.
\end{align*}\]
Proof.

\[ K \# J_n = \text{2n crossings} \]

\[ = \]

On Sharp Moves
Local moves generated by a $\#$ move

(1) 

(2) 

(3) 

(4)
$K, K'$: oriented knot
$d_G^\#(K, K')$: the minimum number of $\#$ moves making $K$ into the $K'$

**Theorem.** $K, K'$: oriented knot
$d_G^\#(K, K')=1 \iff \text{Arf}(K) \neq \text{Arf}(K')$

**Remark.**
$d_G^\#(K, \bigcirc)=u_\#(K)$
Example.

\[
\begin{align*}
\text{Arf}(5_1) &= 1 \\
\text{Arf}(O) &= 0 \\
\text{Arf}(3_1) &= 1 \\
\text{Arf}(5_2) &= 0 \\
\text{Arf}(4_1) &= 1
\end{align*}
\]
Main theorem.
Theorem [H. Murakami-S. Sakai (1993)]

\( K \): trefoil knot, \( J_n \): 2-bridge knot \( C(4,2n) \),
\( u_\#(K) = 1 \)

\( \implies u_\#(K \# J_n) = 1 \).
Let $n$ be 3 or an even integer. Then there exist two infinite families of knots $J_1, J_2, \ldots, J_p, \ldots$ and $K_1, K_2, \ldots, K_q, \ldots$ such that:

1. $J_p, K_q$: $n$-bridge knots,
2. $u_{\#}(J_p \# K_q) = 1$ ($p, q = 1, 2, \ldots$).
$J_p, \ K_q \ (n = 2)$

$J_p: \ C(-4, 2p - 1) \quad K_q: \ C(4, -2q),$

\[ \text{Theorem.} \quad p = 1 \implies \text{Theorem[H. Murakami-S. Sakai]} \]
$J_p, \ K_q \ (n = 3)$
$J_p, K_q: (n = 4)$
Proof of the theorem.
Lemma
Proof of main theorem \((n = 2)\)
Let $n, m$ be 3 or an even integer. Then there exist two infinite families of knots $J_1, J_2, \ldots, J_p, \ldots$ and $K_1, K_2, \ldots, K_q, \ldots$ such that:

1. $J_p, : n$-bridge knots,
2. $K_q, : m$-bridge knots,
3. $u_\#(J_p \# K_q) = 1$ ($p, q = 1, 2, \ldots$).
2-bridge knot ≠ 4-bridge knot

\[ K_q \]

\[ J_p \]

\[ 2q \]

\[ 2p-1 \]